

The Vacuum Catastrophe and the Granularity of Hilbert Space

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Quantum field theory with a Planckian cutoff predicts a vacuum energy density of $4.6 \times 10^{113} \text{ J m}^{-3}$, 123 orders of magnitude above what the observed cosmological constant allows. This paper resolves the discrepancy in two steps, within a framework in which every quantum state carries the same finite budget of L bits and gravity follows from that limit. First, the zero-point energy does not gravitate: a stress tensor proportional to the metric has no flux through any null surface, so it never enters the thermodynamic derivation of Einstein's equations, and Λ survives there as an integration constant that the horizon fixes. Second, the error in the standard estimate is in the state count. The energy Λ carries inside the horizon is the horizon entropy times the horizon temperature, which gives $\rho_\Lambda = 3\pi\rho_{\text{Planck}}/2L^2$ with $L = 6.4 \times 10^{61}$ the number of Planck lengths around the horizon: one factor of L because the count is of a boundary and not of a volume, and one because the energy scale of a pixel is E_P/L and not E_P . The acceleration scale of galactic rotation curves, $a_0 = (1.20 \pm 0.24) \times 10^{-10} \text{ m s}^{-2}$, fixes L independently at $(4.6 \pm 0.9) \times 10^{61}$ and gives $\rho_\Lambda = (1.0 \pm 0.4) \times 10^{-9} \text{ J m}^{-3}$, against the observed 5.2×10^{-10} ; in the measured variable a_0 the two agree at 1.4σ . The same L caps a state spread across its Hilbert space at 205 qubits, a third determination with no astronomy in it.

Keywords: Cosmological constant; vacuum energy; emergent gravity; holographic principle; discrete Hilbert space; rational quantum mechanics

I. THE PROBLEM

Quantum field theory assigns every mode of every field a zero-point energy $\frac{1}{2}\hbar\omega$. Summed to a Planckian cutoff this gives a vacuum energy density of order the Planck density,

$$\rho_{\text{vac}} \sim \frac{c^7}{\hbar G^2} = 4.6 \times 10^{113} \text{ J m}^{-3}, \quad (1)$$

while the observed cosmological constant corresponds to

$$\rho_\Lambda = \frac{\Lambda c^4}{8\pi G} = 5.2 \times 10^{-10} \text{ J m}^{-3}, \quad (2)$$

a ratio of 8.8×10^{122} , or in Planck units $\Lambda \ell_P^2 = 2.85 \times 10^{-122}$ [1, 2]. Adler, Casey and Jacob named this the vacuum catastrophe [3], after the ultraviolet catastrophe it resembles. The cutoff is not the issue. Electroweak symmetry breaking and the QCD condensate contribute at $(200 \text{ GeV})^4$ and $(0.3 \text{ GeV})^4$, some 10^{56} and 10^{45} times the observed value, and no symmetry protects their sum [2]. General relativity requires every form of energy to gravitate [4], so either the vacuum energy is canceled to 123 decimal places or it is not a source of curvature.

This paper takes the second route, within the framework of Ref. [5], in which Hilbert space is granular with a single universal integer L and gravity is derived from that granularity. A uniform vacuum energy is not a source, because a stress tensor proportional to the metric has no flux through a null surface (Sec. III); this removes the $10^{113} \text{ J m}^{-3}$ but leaves Λ as an unexplained integration

constant. The scale of what remains is set by the number of degrees of freedom a horizon volume contains, which is L^2/π and not $(R/\ell_P)^3$, and that factor is the catastrophe (Sec. IV). Sections V and VI fix L from galaxies and from the laboratory.

II. THE FRAMEWORK

Palmer's Rational Quantum Mechanics [6, 7] keeps the Schrödinger equation and the Born rule but admits only those states, and only those measurement bases, for which a qubit $|\psi\rangle = \cos\frac{\theta}{2}|1\rangle + e^{i\phi}\sin\frac{\theta}{2}|{-1}\rangle$ has

$$\cos^2\frac{\theta}{2} = \frac{m}{L}, \quad \frac{\phi}{2\pi} = \frac{n}{L}, \quad (3)$$

with $m \in \{0, \dots, L\}$, $n \in \{0, \dots, L-1\}$, and L a large integer. Such a state is a string of L bits: the fraction of $+1$ bits is $\cos^2(\theta/2)$, and rotating the string by n places is the phase [7]. In Palmer's version L is a property of each qubit and ranges from 10^{64} to 10^{109} [7]. Reference [5] takes L to be one universal integer for all systems (Axiom 1) and identifies one step of quantum phase, $2\pi/L$, with one Planck length on a great circle of the de Sitter horizon (Axiom 2). That horizon is the sphere of radius $R_\Lambda = \sqrt{3/\Lambda} = 1.66 \times 10^{26} \text{ m}$ from beyond which no light will ever reach an observer in a universe whose expansion is driven by Λ . Axiom 2 reads

$$\ell_P = \frac{2\pi R_\Lambda}{L}. \quad (4)$$

The de Sitter radius, rather than the present Hubble radius, is the length in Axiom 2 because L is a constant and R_Λ is the only cosmological length that does not change

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with time. From the two axioms, Verlinde's entropic argument [10] returns $G = c^3 \ell_P^2 / \hbar$ and Jacobson's [11] returns Einstein's equations. The Gibbons-Hawking temperature of the horizon [12] is $k_B T_{\text{dS}} = \hbar c / 2\pi R_\Lambda = E_P / L$, with $E_P = \hbar c / \ell_P$ the Planck energy. And, up to an order-unity coefficient discussed in Sec. V, $a_0 = c^2 / L \ell_P$ is the acceleration below which galactic rotation curves flatten. References [8, 9] restate the two axioms as one postulate, a ring of L Planck cells on which every quantum state is a pattern, closing on a great circle of the horizon in Ref. [8] and generalized to the whole sphere in Ref. [9], from which the three dimensions of space follow. The two quantities used below are the pixel count of the horizon and the energy scale of one pixel.

III. WHY THE ZERO-POINT ENERGY IS NOT A SOURCE

The framework derives Einstein's equations by Jacobson's argument [11]. The Clausius relation $\delta Q = T dS$ is imposed on every local Rindler horizon, the horizon seen by a uniformly accelerating observer, with T the Unruh temperature that observer measures in the vacuum, dS the change in horizon area in Planck units, and δQ the energy flux through the horizon,

$$\delta Q \propto \int \lambda T_{\mu\nu} k^\mu k^\nu d\lambda dA, \quad (5)$$

where k^μ is the null generator of the horizon, the light ray that sweeps it out, and λ its affine parameter. A stress tensor proportional to the metric has no such flux, since $g_{\mu\nu} k^\mu k^\nu = 0$ for null k^μ . The zero-point energy, $T_{\mu\nu} = -\rho_{\text{vac}} g_{\mu\nu}$, therefore never enters the derivation, whatever ρ_{vac} may be. Nor do the electroweak and QCD condensates, which are Lorentz-invariant and of the same form. Lorentz invariance is what fixes that form: a cutoff on three-momentum breaks it and gives the zero-point sum a radiation-like pressure, but that pressure is an artifact of the regulator, and any covariant regularization returns $T_{\mu\nu} \propto g_{\mu\nu}$ [2]. What Jacobson's argument returns is

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}, \quad (6)$$

with Λ an integration constant [11, 13]. Nothing in Eq. (6) prevents that constant from being set equal to $8\pi G \rho_{\text{vac}} / c^4$, which would put the zero-point energy back by hand. What prevents it here is Axiom 2, which fixes Λ from the horizon and not from the field content. The same holds at the Newtonian level in Verlinde's derivation [10]: the force comes from the change in screen entropy as a mass is displaced, and a background that is the same on both sides of the screen produces none.

These are the two conditions Padmanabhan [14] identifies for an emergent theory to be immune to the zero-point energy: the field equations are unchanged by $L_{\text{matter}} \rightarrow L_{\text{matter}} + \text{const}$, and Λ enters solutions as

an integration constant rather than as a parameter in the action. Both hold in unimodular gravity, where the trace-free field equations are blind to a constant shift of $T_{\mu\nu}$ [15, 16]. Here they are not adopted; they are how gravity was derived.

What drops out is the part of the energy density that is the same at every point. Vacuum *differences* have flux through a null surface and gravitate. The vacuum-loop correction to the electrostatic energy of a nucleus, the same physics as the Lamb shift, is a shift in binding energy relative to the surrounding vacuum, localized and different for aluminum and platinum, and free-fall tests of the equivalence principle show that it weighs, to one part in 10^6 [17, 18]. So does the proton, most of whose mass is QCD binding energy: a localized excitation above the condensate, not the condensate itself.

Equation (6) settles half of the problem. The zero-point sum of Eq. (1) is proportional to the metric, has no flux through any null surface, and never enters the field equations. Whatever quantum field theory says the vacuum weighs, gravity does not respond to it.

The other half is Λ . It survives in Eq. (6) as an integration constant: the equation does not fix its value, and something outside the equation must. In the present framework the horizon does. Axiom 2 ties the Planck length to the horizon through L , and with $\Lambda = 3/R_\Lambda^2$ this reads

$$\Lambda \ell_P^2 = \frac{12\pi^2}{L^2} = 2.85 \times 10^{-122}. \quad (7)$$

Λ is small in Planck units because the horizon is large in Planck lengths, $L = 6.4 \times 10^{61}$ of them around. The value of L is measured independently in Sec. V. What remains to be said here is what the energy of Λ physically is, and why it comes to 10^{-9} J m^{-3} rather than the 10^{113} of the zero-point sum. The next section answers by counting the degrees of freedom on the horizon.

IV. THE STATE COUNT

The continuum estimates the energy of the vacuum from the bulk. A region of radius R is assigned $(R/\ell_P)^3$ zero-point modes at the Planck energy apiece, and spread over the volume $\sim R^3$ these give ρ_{Planck} for any R ; that is Eq. (1). Section III showed that none of it gravitates. What does gravitate is the energy of Λ , and the framework locates it on the horizon. The derivation is five steps.

Step 1. Boundary count. Axiom 2 gives $R_\Lambda = L \ell_P / 2\pi$. The horizon is a two-sphere, because space has three dimensions, which Ref. [9] derives from the same L . Its budget of degrees of freedom is its pixel count,

$$N_{\text{pix}} = \frac{4\pi R_\Lambda^2}{\ell_P^2} = \frac{L^2}{\pi} = 1.32 \times 10^{123}, \quad (8)$$

an area rather than a volume. Counted in the register's own cells, which have area ℓ_P^2 / π on the horizon [9], the

sphere has L^2 cells instead; the two counts differ by π and give the same entropy, $L^2/4\pi$ in units of k_B . The Planck pixel is the cell of the Bekenstein–Hawking entropy and is the one used here.

Step 2. Energy scale of a pixel. The Gibbons–Hawking temperature [12] of the horizon, $k_B T_{\text{dS}} = \hbar c / 2\pi R_\Lambda$, with $2\pi R_\Lambda = L\ell_P$ from Axiom 2, sets the energy scale of one pixel,

$$\varepsilon = k_B T_{\text{dS}} = \frac{\hbar c}{L\ell_P} = \frac{E_P}{L} = 3.0 \times 10^{-53} \text{ J}, \quad (9)$$

the Planck energy divided by L , rather than the Planck energy.

Step 3. Total energy. The Bekenstein–Hawking entropy of the horizon is $S_{\text{BH}} = k_B A / 4\ell_P^2 = \frac{1}{4} N_{\text{pix}} k_B$, and its thermal energy is

$$E = S_{\text{BH}} T_{\text{dS}} = \frac{1}{4} N_{\text{pix}} \varepsilon = 1.00 \times 10^{70} \text{ J}. \quad (10)$$

This is the energy Λ carries inside the horizon, $\rho_\Lambda \cdot \frac{4}{3}\pi R_\Lambda^3$, exactly. Equivalently, since for $p = -\rho$ the Komar energy, the mass that sources gravity in general relativity, is $2\rho_\Lambda V$ in magnitude,

$$2\rho_\Lambda V = \frac{1}{2} N_{\text{pix}} \varepsilon, \quad (11)$$

one half-quantum $\frac{1}{2} k_B T_{\text{dS}}$ per pixel, which is Padmanabhan’s holographic equipartition [14, 19] applied to the de Sitter horizon.

Step 4. Energy density. Dividing by the volume $V = \frac{4}{3}\pi R_\Lambda^3 = \frac{4}{3}\pi (L\ell_P/2\pi)^3$,

$$\rho_\Lambda = \frac{E}{V} = \frac{3\pi}{2} \frac{E_P}{L^2 \ell_P^3}. \quad (12)$$

Step 5. In Planck units. With $\rho_{\text{Planck}} = E_P / \ell_P^3 = c^7 / \hbar G^2 = 4.63 \times 10^{113} \text{ J m}^{-3}$,

$$\boxed{\frac{\rho_\Lambda}{\rho_{\text{Planck}}} = \frac{3\pi}{2L^2} = 1.13 \times 10^{-123}} \quad (13)$$

and $\rho_\Lambda = 5.25 \times 10^{-10} \text{ J m}^{-3}$. As an energy scale this is $\rho_\Lambda^{1/4} = (3\pi/2)^{1/4} E_P / \sqrt{L} = 2.2 \text{ meV}$, the millielectronvolt of dark energy written in the framework’s constants. One factor of L is lost to the dimensionality of the count, Step 1, and one to the energy scale of a pixel, Step 2.

Steps 3 and 4 are identities of de Sitter thermodynamics and hold in general relativity for any Λ : the thermal energy of a de Sitter horizon, spread over its interior, is the Λ that produces the horizon. Steps 1, 2 and 5 use Axiom 2, and given a value of L from any source they predict the pixel count, the energy scale of a pixel and the vacuum energy density. What Axiom 1 adds is that the integer fixing the number of states inside the horizon, whose logarithm is the entropy $L^2/4\pi$ of Step 3, is the same L that bounds every quantum state [8]. The horizon count is then not merely an area in Planck units but the information capacity of the universe, which is Banks’

proposal that a positive Λ and a finite Hilbert space go together [20] with its integer named. With L taken from the horizon, as here, Eq. (13) returns the observed density by construction; the test is Sec. V, where L comes from galaxies.

The exponent is robust. $L = 6.4 \times 10^{61}$ is read off a known length: one does not need Λ to know that a cosmological horizon is some 10^{26} m in radius, and the exponent does not depend on which horizon is meant. The Hubble radius gives $(R/\ell_P)^{-2} = 10^{-121.9}$, the de Sitter radius $10^{-122.0}$, the particle horizon $10^{-122.9}$, a spread of one decade across lengths differing by a factor of three, against a problem of 123 decades. The scaling $\rho_\Lambda \sim \rho_{\text{Planck}} (\ell_P/R)^2$ has been reached before, from effective field theory [21] and from mode counting [22–24]. Here it follows from the two axioms rather than from an assumed relation between cutoffs, and L is fixed by two measurements those derivations do not have: the galactic acceleration scale (Sec. V) and the qubit ceiling (Sec. VI).

The 123 orders of magnitude are therefore the square of the number of Planck lengths around the horizon. The continuum’s estimate uses $(R_\Lambda/\ell_P)^3 = (L/2\pi)^3$ bulk modes at E_P ; the energy that gravitates is L^2/π boundary pixels at the scale E_P/L ; and the ratio is L^2 , up to a numerical factor.

V. L FROM GALAXIES

Reference [5] fixes L a second time, from data with no cosmological content. The acceleration scale of the radial acceleration relation, $a_0 = (1.20 \pm 0.02_{\text{stat}} \pm 0.24_{\text{sys}}) \times 10^{-10} \text{ m s}^{-2}$, is measured from galactic rotation curves [25], the systematic term coming from the stellar mass-to-light ratio. Then $a_0 = c^2/L\ell_P$ inverts to $L_{\text{gal}} = c^2/a_0\ell_P = (4.6 \pm 0.9) \times 10^{61}$, and carried through Eqs. (4) and (13) this gives

$$\Lambda_{\text{pred}} = \frac{12\pi^2}{\ell_P^2 L_{\text{gal}}^2} = \frac{12\pi^2 a_0^2}{c^4} = (2.1 \pm 0.8) \times 10^{-52} \text{ m}^{-2}, \quad (14)$$

or in the units of Eq. (1) a vacuum energy density of $(1.0 \pm 0.4) \times 10^{-9} \text{ J m}^{-3}$. The observed values are $1.09 \times 10^{-52} \text{ m}^{-2}$ [26] and $5.2 \times 10^{-10} \text{ J m}^{-3}$. Prediction and observation agree. In the measured variable, the horizon value of L corresponds to $a_0 = 8.6 \times 10^{-11} \text{ m s}^{-2}$, 1.4σ from the measured value, and the 123 decades of Eq. (1) are closed from rotation curves alone. Rotation curves and cosmological surveys are unrelated measurements.

Milgrom noticed in 1983 that $a_0 \sim cH_0$ [27]. Equation (14) turns the coincidence into an equation, $\Lambda \propto a_0^2/c^4$, whose coefficient the framework fixes up to one order-unity factor. That factor is the coefficient γ in $a_0 = \gamma c^2/L\ell_P$, which Ref. [5] does not derive and Refs. [8, 9] leave open. The coefficient is an open problem in emergent gravity itself: Verlinde’s derivation gives $cH_0/6 = 1.1 \times 10^{-10} \text{ m s}^{-2}$ [28], and fits of rotation curves within emergent gravity prefer a scale about 30%

lower [29, 30]. The 2π in Axiom 2 itself is not free; it is the number of Planck cells around a great circle, which Ref. [9] checks against the horizon’s independently defined pixels. The measured a_0 gives $\gamma = 1.39 \pm 0.28$. The framework’s own scale, $\gamma = 1$, lies at 1.4σ ; $\gamma = 1/2$ at 3.2σ ; and the bare surface gravity of the horizon, c^2/R_Λ , at 18σ . The remaining factor of 1.9 between Eq. (14) and the observed Λ is γ^2 .

VI. TEST

A state of N qubits spread across its Hilbert space has 2^N outcomes of nonzero probability. In the framework each probability is a frequency over L trials, hence at least $1/L$, and the probabilities sum to one; so $2^N \leq L$, and

$$N_{\max} = \lfloor \log_2 L \rfloor = 205, \quad (15)$$

or 204 at the galactic value of L [8]. This is Palmer’s $\log_2 L$ [7]; his bit count $2^{N+1} - 2 \leq NL$, which gives 212 and which Ref. [5] called exact, is a weaker necessary condition [8]. Beyond $N_{\max}$, algorithms that spread amplitude across the whole of Hilbert space lose their exponential advantage. Operationally, a random circuit on N fault-tolerant logical qubits followed by its inverse returns to its initial state with the gate fidelity in standard quantum mechanics, and here with probability only about $(L/2^N)^2$ once N exceeds 205 [8]. The ceiling is technology-independent, the same for superconducting qubits as for trapped ions, which distinguishes it from every known decoherence mechanism. A measured $N_{\max}$ would fix L from the laboratory to a factor of two, $2^{N_{\max}} \leq L < 2^{N_{\max}+1}$, and with it the exponent in Eq. (13). If devices carry states spread across well over 205 logical qubits with no such wall, the account given here fails at its premise.

The framework also requires Λ to be constant, since L is. Reference [5] turns the lunar-laser-ranging bound on $\dot{G}/G$ [31] into $|1 + w_0| \lesssim 5 \times 10^{-4}$ for the present dark-energy equation of state. DESI DR2 prefers evolving dark energy over a constant Λ at 3.1σ with the cosmic microwave background, and at 2.8σ to 4.2σ with supernovae added, depending on the sample [32]. A confirmed time variation of the dark-energy density would falsify the identification of R_Λ with a fixed length in Axiom 2; the framework predicts that the preference will not survive.

VII. CONCLUSION

The vacuum catastrophe has two parts. The zero-point energy never enters the field equations, because in an emergent theory those equations are unchanged by a constant shift of the matter Lagrangian and Λ appears as a boundary condition. The scale of that boundary condition is set by the state count. The continuum assigns a horizon volume $(R/\ell_P)^3$ modes at E_P apiece and returns the

Planck density for any R . A universe of finite information has L^2/π pixels on its horizon, each at the energy scale E_P/L , and the energy Λ carries inside the horizon is $S_{\text{BH}}T_{\text{dS}}$. Once a quantum state cannot hold unlimited information, $\rho_\Lambda/\rho_{\text{Planck}} = 3\pi/2L^2$ is the only number the counting can give. L has been determined at the horizon and from galactic rotation curves, and the two agree, at 1.4σ of the galactic measurement. A third determination, with no astronomy in it, is a ceiling of 205 usefully entangled qubits.

Appendix A: Numerical inputs

$c = 2.99792458 \times 10^8 \text{ m s}^{-1}$, $\hbar = 1.0546 \times 10^{-34} \text{ J s}$, $G = 6.6743 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$, $\ell_P = 1.6163 \times 10^{-35} \text{ m}$, $E_P = \hbar c/\ell_P = 1.956 \times 10^9 \text{ J}$; $\Lambda = 1.09 \times 10^{-52} \text{ m}^{-2}$ from $H_0 = 67.4 \text{ km s}^{-1} \text{ Mpc}^{-1}$ and $\Omega_\Lambda = 0.685$ [26]; $a_0 = (1.20 \pm 0.02_{\text{stat}} \pm 0.24_{\text{sys}}) \times 10^{-10} \text{ m s}^{-2}$ [25]. Uncertainties are propagated linearly from the systematic term of a_0 .

TABLE I. Derived quantities.

Quantity	Expression	Value
ρ_{Planck}	$c^7/\hbar G^2$	$4.63 \times 10^{113} \text{ J m}^{-3}$
ρ_Λ	$\Lambda c^4/8\pi G$	$5.25 \times 10^{-10} \text{ J m}^{-3}$
R_Λ	$\sqrt{3/\Lambda}$	$1.659 \times 10^{26} \text{ m}$
L	$2\pi R_\Lambda/\ell_P$	6.449×10^{61}
L_{gal}	$c^2/a_0\ell_P$	$(4.6 \pm 0.9) \times 10^{61}$
γ	$a_0 L \ell_P / c^2$	1.39 ± 0.28
N_{pix}	L^2/π	1.324×10^{123}
N_{cell}	L^2	4.159×10^{123}
S_{BH}/k_B	$L^2/4\pi$	3.310×10^{122}
Bulk modes	$(R_\Lambda/\ell_P)^3$	1.08×10^{183}
ε	E_P/L	$3.03 \times 10^{-53} \text{ J}$
$\rho_\Lambda/\rho_{\text{Planck}}$	$3\pi/2L^2$	1.133×10^{-123}
$\rho_\Lambda V$	$S_{\text{BH}}T_{\text{dS}}$	$1.004 \times 10^{70} \text{ J}$
$\rho_\Lambda^{1/4}$	$(3\pi/2)^{1/4} E_P/\sqrt{L}$	2.24 meV
$N_{\max}$	$\lfloor \log_2 L \rfloor$	205

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